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TECHNICAL NOTE No. 900

On Jones' Criterion For Thin Wings Of Minimum Drag

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DEPARTMENT OF THE ARMY PROJECT No. 503-03-001
ORDNANCE RESEARCH AND DEVELOPMENT PROJECT No. TB3-0108

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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MAY 1954

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ON JONES' CRITERION FOR THIN WINGS OF MINIMUM DRAG

In reference 1 Jones has derived necessary conditions for minimum drag shapes of thin wings subject to conditions of given lift, given maximum drag or given volume. In deriving these conditions, the concept of the "combined flow field" was used. This consists of superimposing the flow fields of the forward and reversed motions. In this note, it is shown how these conditions may be derived using the standard methods of calculus of variations and the general reverse flow theorem.[2] The method will be illustrated for the case of given lift. In addition it is shown that the necessary condition actually yields a minimum for the drag. Linearised theory is assumed throughout.

Let p and α denote the local lift and angle of attack distributions in forward flow. These are defined on the projection of the wing surface on a mean plane. This area is denoted by S and all integrations are over this area. As in reference 3, two types of reverse flow are considered. In the first, the lift in reverse flow $\bar{p} = p$, and $\bar{\alpha}$ is determined from linearised theory. In the second, the angle of attack in reverse flow $\tilde{\alpha} = -\alpha$, and $\tilde{p}$ is determined from linearised theory. The reverse flow theorem states that

$$\int p\, \bar{\alpha}\, d S = \int \bar{p}\, \alpha\, d S \tag{1}$$

and similarly for the second type. Since p and α are related linearly, $p + \epsilon q$ must correspond to $\alpha + \epsilon \beta$ where β is the angle of attack distribution corresponding to the lift q. Defining the variation of p,

$$\delta p = \epsilon q$$

then the variation of α is

$$\delta \alpha = \epsilon \beta$$

The reverse flow theorem can, of course, be applied to δp and $\delta\alpha$.

Consider now the variational problem of minimizing the drag, D, subject to condition of a given lift L. Since

$$D = \int p\, \alpha\, d S \tag{2}$$

$$L = \int p\, d S \tag{3}$$

the quantity $D + \lambda L$, where λ is an undetermined constant multiplier, should be minimised according to the rules of the calculus of variations. Then

$$\delta \ (D + \lambda L) = \int (p \ \delta \ \alpha + \alpha \ \delta \ p + \lambda \ \delta \ p) \ d \ S \tag{4}$$

Since $\overline{p} = p$, the first term of the integrand can be written

$$\int p \ \delta \ \alpha \ d \ S = \int \overline{p} \ \delta \ \alpha \ d \ S$$
$$= \int \overline{\alpha} \ \delta \ p \ d \ S$$

Thus (4) can be written

$$\delta \ (D + \lambda L) = \int (\overline{\alpha} + \alpha + \lambda) \ \delta \ p \ d \ S$$

Since the variation of $D + \lambda L$ must be zero for arbitrary δp, the necessary condition for minimum drag with given lift is

$$\alpha + \overline{\alpha} = - \lambda = \text{Constant}$$

which is Jones' criterion. Multiplying the last equation by p and integrating determines λ, so that

$$\alpha + \overline{\alpha} = \frac{2 D}{L} \tag{5}$$

The criterion can be expressed in a different form involving the second type of reverse flow. Let the flat plate of unit angle of attack be denoted by α_f, i.e. $\alpha_f = 1$. The lift can be written

$$L = \int p \ d \ S = \int \alpha_f \ p \ d \ S$$
$$= - \int \widetilde{\alpha}_f \ p \ d \ S$$
$$= - \int \widetilde{p}_f \ \alpha \ d \ S$$

Taking the first variation as in (4) yields

$$\delta \ (D + \lambda L) = \int (p - \widetilde{p} - \lambda \ \widetilde{p}_f) \ \delta \ \alpha \ d \ S$$

and since $\delta \alpha$ is arbitrary

$$p - \widetilde{p} = \lambda \ \widetilde{p}_f$$

where now

$$\lambda = - \frac{2 D}{L}$$

The question of what type of functions p and α are admissible in the variational process needs to be discussed. It is known that the reverse flow theorem, eq. (1), does not hold if the lift has a leading edge singularity in forward flow, since the wing in reverse flow does not satisfy the Kutta condition at the trailing edge. Therefore admissible functions

p must be such that they do not have leading edge singularities. However there are indications that Jones' criterion is valid even for functions p which have these singularities[3].

Using the concept of orthogonal loadings [3,4] it can be shown that Jones' criterion is also sufficient, that is, it actually yields a minimum. Let α_1 and α_2 be two angle of attack distributions and p_1 and p_2 the corresponding lifts[2]. Define

$$(\alpha_1, \alpha_2) = \frac{1}{2} \int (\alpha_1 p_2 + \alpha_2 p_1) \, d\,S \tag{6}$$

i.e., (α_1, α_2) is one-half the mutual interference drag of the two distributions. Then α_1 is orthogonal to α_2 if $(\alpha_1, \alpha_2) = 0$. Since

$$\begin{aligned}(\alpha_1, \alpha_1) &= \int p_1 \alpha_1 \, d\,S \\ &= D_1 \geq 0\end{aligned}$$

it is easy to show that Schwarz's inequality holds

$$(\alpha_1, \alpha_2)^2 \leq (\alpha_1, \alpha_1)\,(\alpha_2\ \alpha_2) \tag{7}$$

Now let α_o be any α such that

$$L_o = \int p_o \, d\,S = 0$$

Then multiplying (5) by p_o, integrating, and applying (1) yields

$$(\alpha, \alpha_o) = 0$$

so that any α with zero total lift is orthogonal to the optimal α, i.e., the α which satifies (5). The converse is also true. If α' is any α such that

$$\int p' \, d\,S = \int p \, d\,S = L$$

then $\alpha - \alpha' = \alpha_o$ since

$$\int (p - p') \, d\,S = 0$$

Thus

$$(\alpha, \alpha - \alpha') = 0$$

or

$$(\alpha, \alpha') = (\alpha, \alpha)$$

Using (7)

$$(\alpha, \alpha)^2 = (\alpha, \alpha')^2 \leq (\alpha,\alpha)\ (\alpha', \alpha')$$

$$(\alpha,\alpha) \leq (\alpha', \alpha')$$

Thus it is shown that of all admissible functions with total lift L, the angle of attack distribution satisfying (5) yields the least drag.

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